IFAS CSIR NET

GEOMETRY OF SHAPES

Q 1 3 unbiased coins are tossed. What is the probability of getting at most two heads?

HHH, HTT, THT, .

(A) $\frac{3}{4}$ (B) $\frac{3}{8}$ (3) $\frac{1}{4}$ (4) $\frac{7}{8}$ ✓

Sol: P("at most two heads") $= \frac{7}{8}$

$2 \times 2 \times 2 = 8$

TT

H, T, T

H, H, T, ...

HHH ✓

Q.2 Four babies born in the month of April 2019 are randomly selected. The probability that atleast two of them will have the same date of birth is?

(1) 81.2% (2) 18.8% (3) 13.7% (4) 86.3%

0.188 × 100%

= 18.8%

Sol^n: "none of them will have same date of birth"

Toss

$P(H) = \frac{1}{2}$

$P(T) = 1 - \frac{1}{2} = \frac{1}{2}$

B_1 B_2 B_3 B_4

Mahima → $\frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30}$ → "none of them have same date of birth"

$= \frac{21924}{27000}$

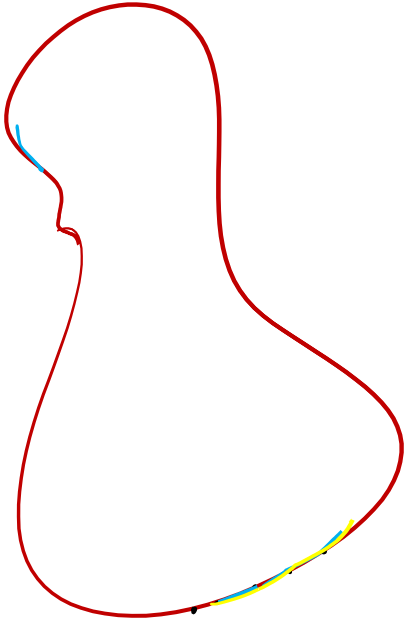
P("atleast two of them have same date of birth")

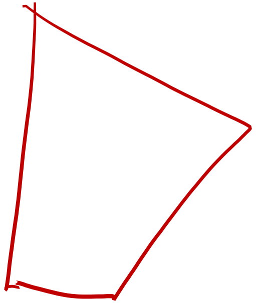
$= 1 - \frac{21924}{27000} = \frac{5076}{27000} = 0.188$

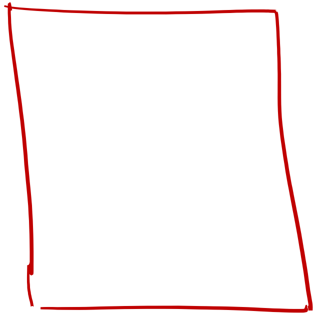
$\frac{29 \times 28}{812}$, $\frac{812 \times 27}{21924}$

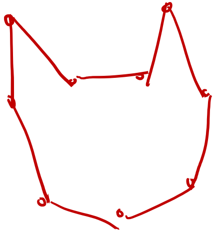
$27000 - 21924 = 5076$

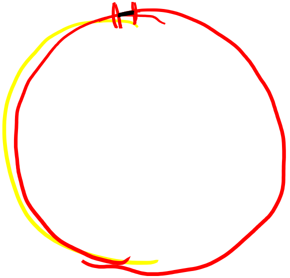
Geometry







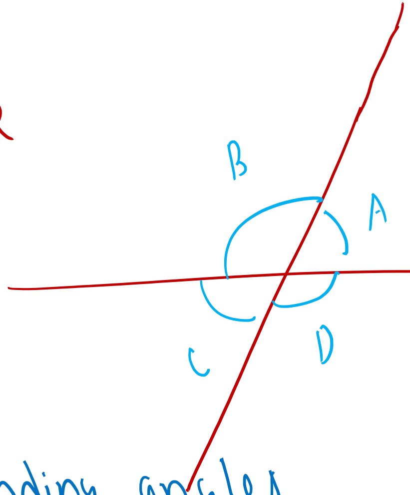




??

* Angle

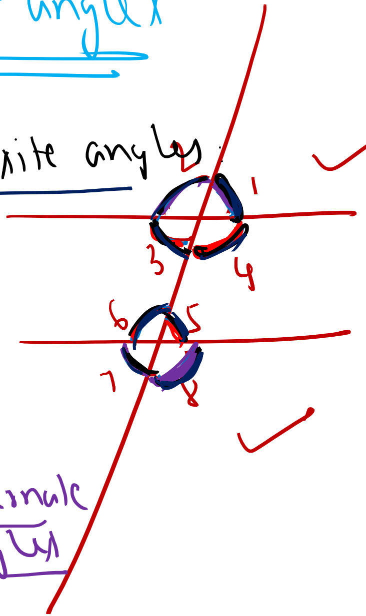
Birth of angles



* Vertically opposite angles

$\angle A = \angle C$

$\angle B = \angle D$



* Corresponding angles

$\angle 1 = \angle 5$

$\angle 2 = 6$

$\angle 4 = \angle 8$

$\angle 3 = \angle 7$

* Interior Alternate angles

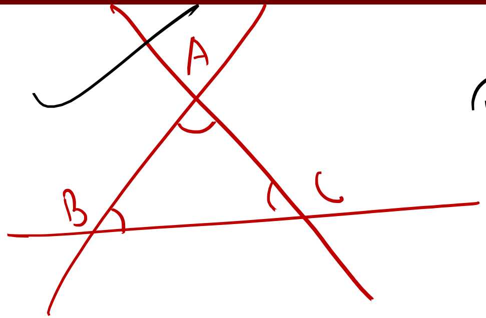
$\angle 4 = \angle 6$

$\angle 3 = \angle 5$

* External Alternate Angles

$\angle 1 = \angle 7$

$\angle 2 = \angle 8$



Triangles

Euclid

300 BC

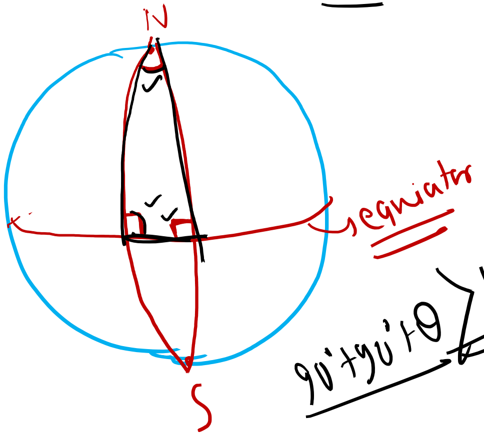
Euclid's Element

Angle: (i) $\angle A + \angle B + \angle C = 180^\circ$

Sides → (ii) sum of any two sides > third side

Bernhard Riemann

1800 Riemannian Geometry



$90^\circ + 90^\circ + \theta > 180^\circ$

** The angles opposite to equal sides are equal.

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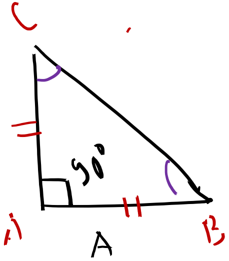
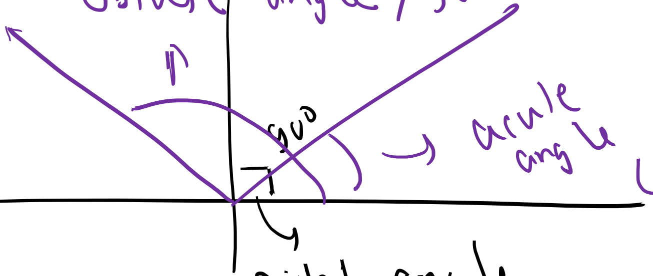
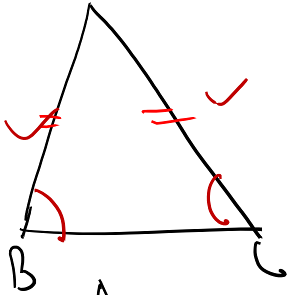


Right angle triangle

$AB = AC$

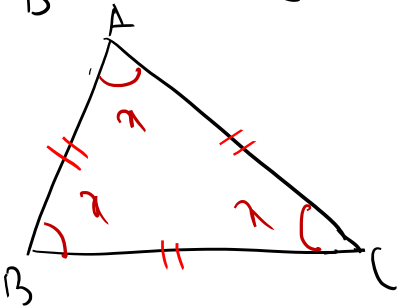




$AB = AC$

Isosceles Triangle

$\angle B = \angle C$



$AB = BC = AC$

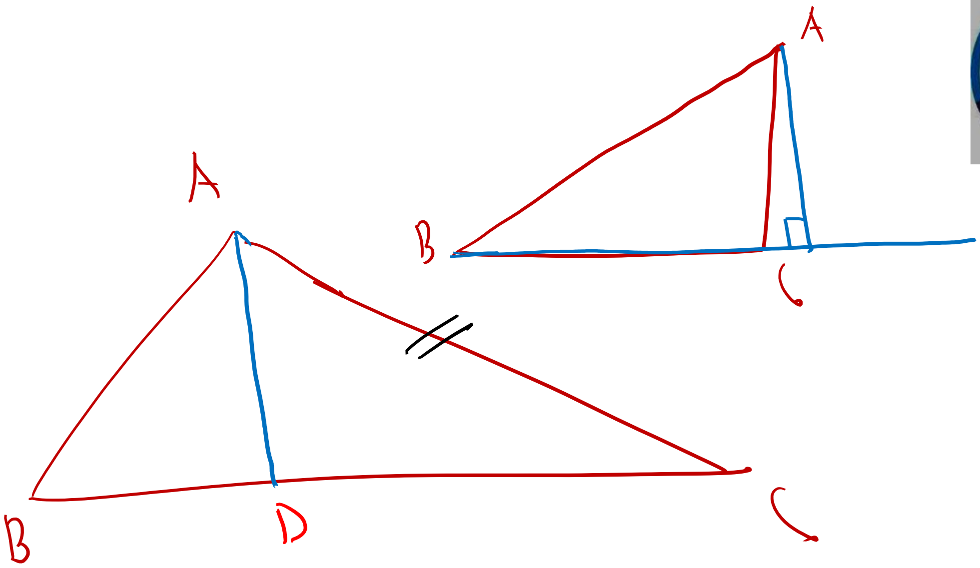
Equilateral Triangle

$\angle A = \angle B = \angle C = 60^\circ$

$x + x + x = 180^\circ$

$\Rightarrow x = 60^\circ$

Area of Triangle:



Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

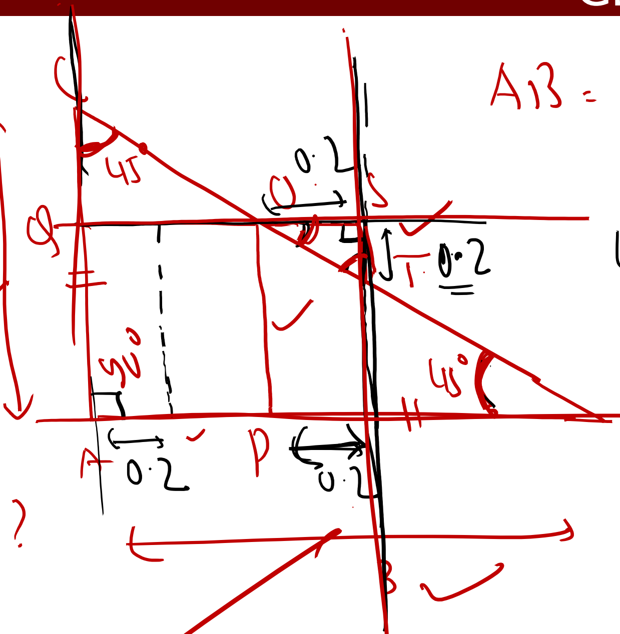
$= \frac{1}{2} \times BC \times AD$

Q.1 (CSIR NET 2014)

* height of APOQ is twice its width

* $\frac{\text{Area of } \Delta ABC}{\text{Area } \Delta OST} = ??$

(1) 625 (2) 400 (3) 225 (4) 125



AB = AC = 3

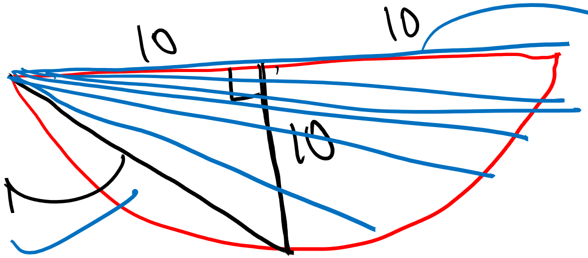
∠SOT = ABC

$$\frac{\text{Area of } \Delta ABC}{\text{Area } \Delta OST} = \frac{\frac{1}{2} \times AB \times AC}{\frac{1}{2} \times OS \times ST} = \frac{\frac{1}{2} \times 3 \times 3}{\frac{1}{2} \times 0.2 \times ST} = 225$$

Q2 (CSIR NET)

What is the length of the largest rod that can be put in a hemispherical bowl of radius 10 cm such that no end of the rod is outside bowl? (Assume that the rod has a negligible thickness)

(1) $10\sqrt{2}$ (2) $10\sqrt{3}$ (3) $10\sqrt{4}$ (4) $10\sqrt{5}$



$\sqrt{200}$

$= 10\sqrt{2}$

$10+10 = 20$

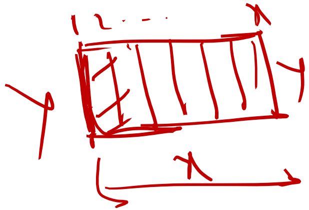
$= 10 \times 2$

$= 10\sqrt{4}$

Q.3 Five congruent rectangles are drawn inside a big rectangle of perimeter 165 as shown. What is the perimeter of one of the 5 rectangles?

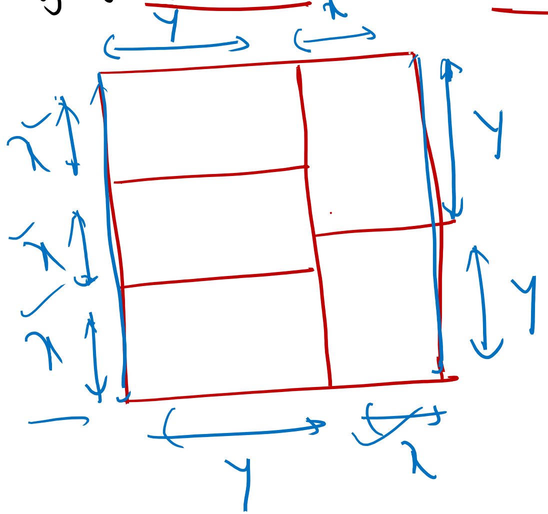
(1) 37 (2) 75

(3) 15 (4) 165



Area = x X Y

x + y + x + y



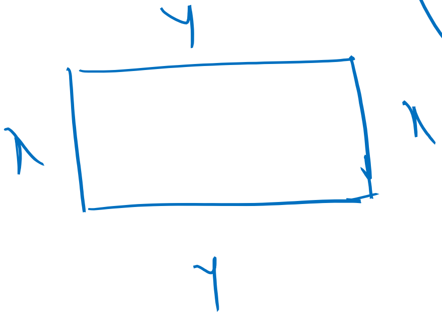
$$5x + 4y = 165 \quad \text{—— (1)}$$

$$3x = 2y \quad , \quad 2y = 45$$

$$5x + 6x = 165$$

$$\Rightarrow 11x = 165$$

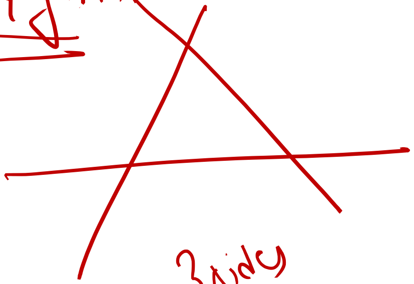
$$\Rightarrow x = 15$$



Perimeter $= 2x + 2y$

$= 30 + 45 = 75$

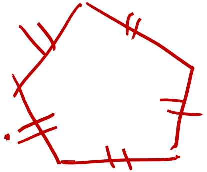
** Polygon: sides



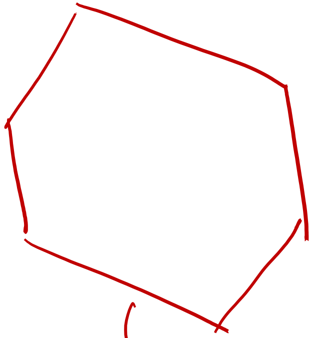
3 sides

4-side

Quad-Laterals



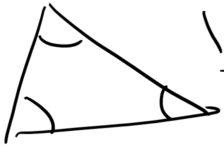
Regular Pentagon



Hexagon

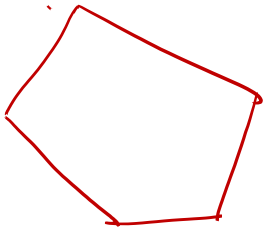
* Sum of interim angles:

n-gon, sum of interior angles $= (n-2) \times 180^\circ$



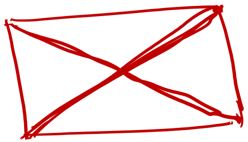
180°

$n=3 \rightarrow (3-2) \times 180^\circ$
$= 1 \times 180^\circ = 180^\circ$



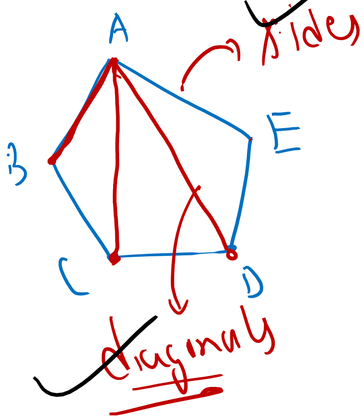
Sum of interior angles
$= (5-2) \times 180^\circ = 540^\circ$

* Number of diagonals:



4 gon

Diagonals = 2



A, B, C, D, E

Adjoining St. lines $= {}^5C_2 = \frac{5!}{3!\,2!} = 10$

Adjoining St. Lines = Sides + Diagonals

Diagonals = 10 − 5 = 5

Diagonals = ${}^5C_2 - 5$

n-gon

The no. of diagonals

$$= {}^{n}C_{2} - n$$

$$= \frac{n!}{(n-2)!\,2!} - n$$

$$= \frac{n(n-1)}{2} - n$$

$$= \frac{n^2 - n - 2n}{2} = \frac{n(n-3)}{2}$$

The no. of diagonals

$$= {}^{n}C_{2} - n$$

$$= \frac{n(n-3)}{2}$$

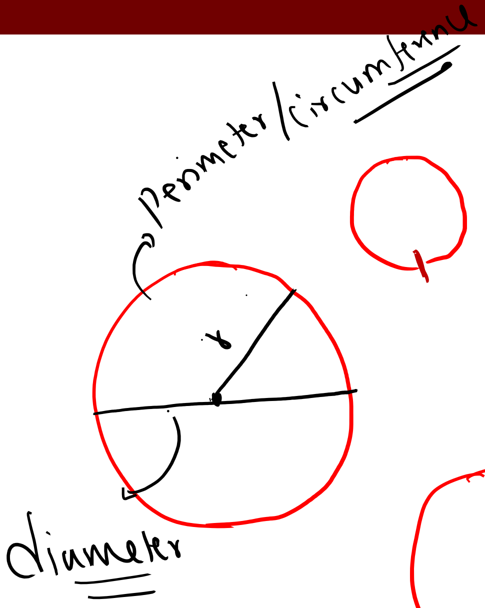
Q3 (CSIR NET Dec 2015)

The number of diagonals of a convex dodecagon (12-gon) is:

(1) 66 (2) 54 (3) 55 (4) 60

$$\frac{n(n-3)}{2} = \frac{12(12-3)}{2} = \underline{54}$$

Circle:



$$\frac{\text{Circumference}}{\text{Diameter}} = \pi \approx 3.141\ldots$$

Diameter $= 2r$

Area $= \pi r^2$

$\frac{1}{2} \times 2\pi r \times r$

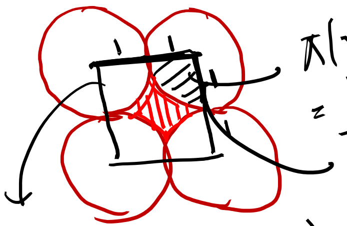
(Circumference $= \pi \times 2r$)

Q. Four circles of unit radius each are drawn such that each one touches two others and their centers lies on the vertex of a square. The area of the region enclosed between the circles is

R.T. Bell

(1) $\pi - 1$ (2) $\pi - 2$

(3) $3 - \pi$ (4) $4 - \pi$

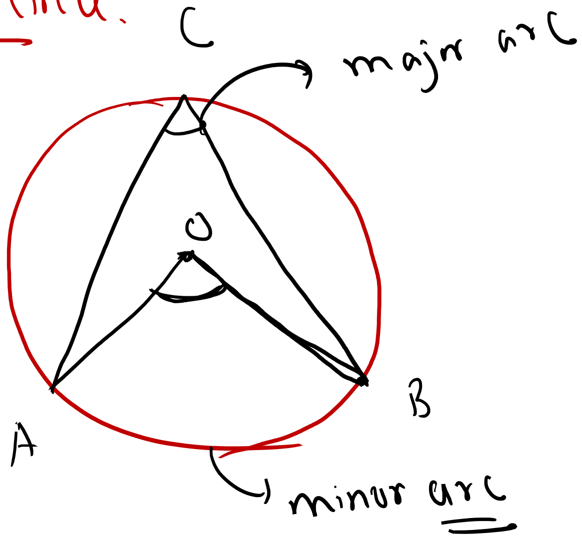


$\pi(r)^2 = \frac{\pi}{4}$

$4 - (4 \times \frac{\pi}{4})$

$= 4 - \pi$

Center theorem of Circle:



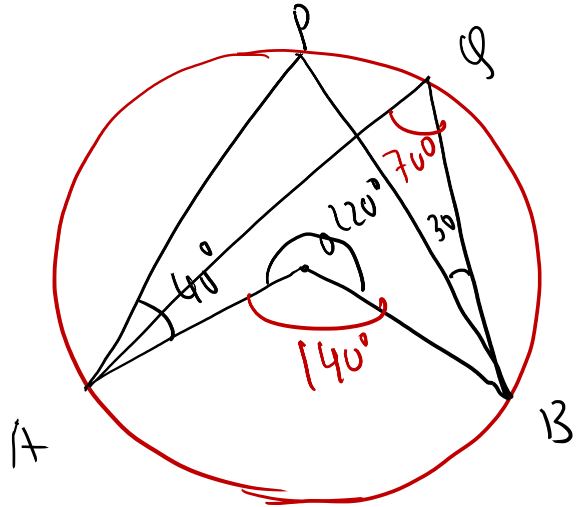
The angle subtended by the arc $\overset{\frown}{AB}$ at center

$\angle AOB = 2\angle ACB$.

$\angle ACB = \frac{1}{2} \times \angle AOB$

(CSIR 2019)

Q



$\angle PAQ = 40°$

$\angle PBQ = 30°$

$360° - 220°$

$= 140°$

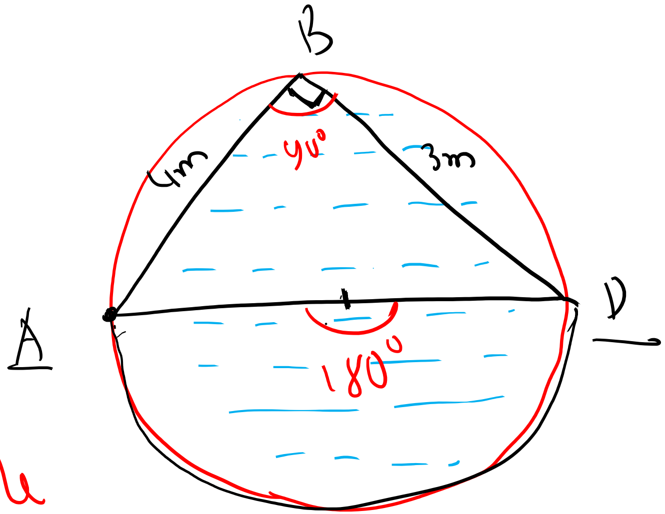
Then $\angle AQB = ?$

(1) 70° (2) 80°

(3) 60° (4) 110°

Q. A turtle starts swimming from a point A located on the circumference of a circular pond. After swimming for 4 meters in a st. line it hits point B on the circumference of the pond. From there it changes direction and swims for 3 meters in a st. line and arrives at a pt. D diametrically opp to point A. How far is the pt D from A?

(1) 3m (2) 4 m (3) 7 m (4) 5 m ✓



B = 90° ??

$AB^2 + BD^2 = AD^2$

$\Rightarrow 16 + 9 = AD^2$

$\Rightarrow AD = \sqrt{25}$

$= 5$

** Diameter - angle theorem:

A angle subtended by the diameter at the circumference will be always 90°.

HAPPY LEARNING

THANKS