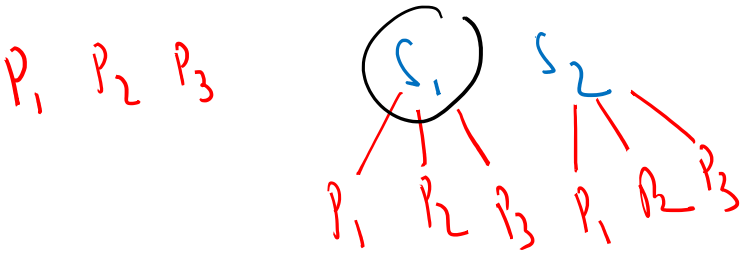
IFAS CSIR NET

PERMUTATIONS AND COMBINATIONS

Fundamental Principle of Counting:

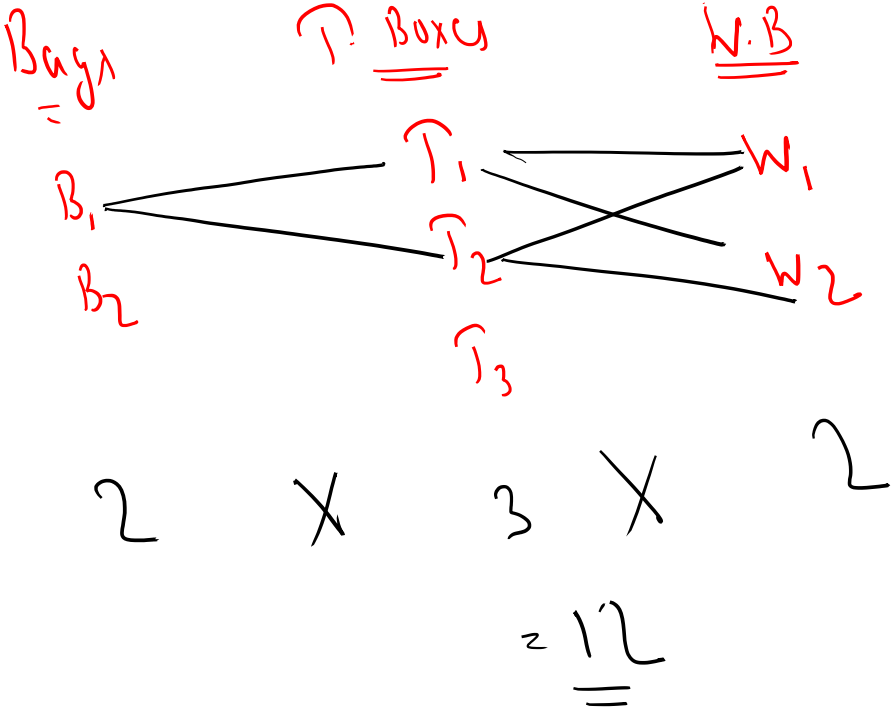
Mohan has 3 pants and 2 shirts. How many different pairs of a pant and a shirt, can he dress up with?



3 + 3

2 X 3 = 6

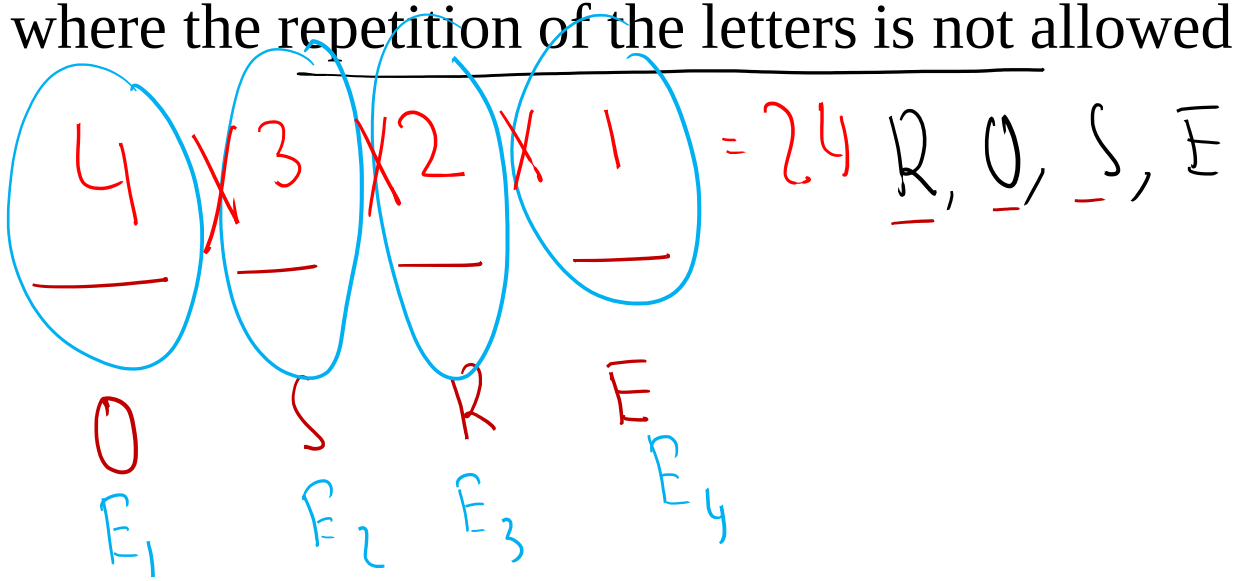
Sabnam has 2 school bags, 3 tiffin boxes and 2 water bottles. In how many ways can she carry these items (choosing one each)?



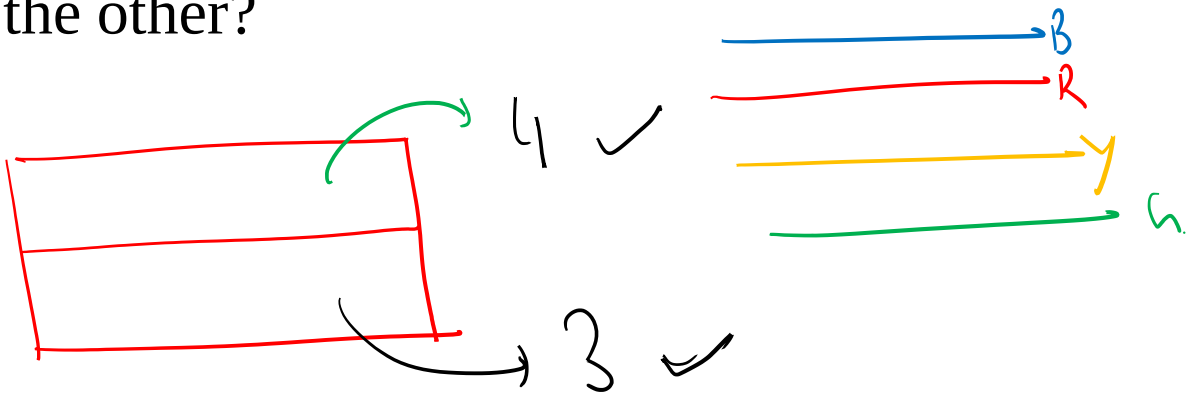
fundamental principle of counting, or, simply, the multiplication principle,

"If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$"

Example 1. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed.



Example 2: Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?



total = 4 x 3 = 12

How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that

(i) repetition of the digits is allowed?
(ii) repetition of the digits is not allowed?

1, 2, 3, 4, 5

(i) $5 \times 5 \times 5 = 125$

(ii) $5 \times 4 \times 3 = 60$

Permutations

Definition 1 A permutation is an arrangement in a definite order of a number of objects (taken some or all at a time).

A, B, C, D, E

1, 2, 3, 4, 5

5 places

=) 5 distinct objects, you need to place them in 3 places. How many different arrangements you can get?

$^5P_3$

$5 \times 4 \times 3 = 60$

$^5P_5 = 5!$

$5 \times 4 \times 3 \times 2 \times 1 = 120$

$${}^{5}P_{3} = 5 \times 4 \times 3 = 60$$

$$5 \times 4 \times 3 = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$= \frac{5!}{2!} = \frac{5!}{(5-3)!}$$

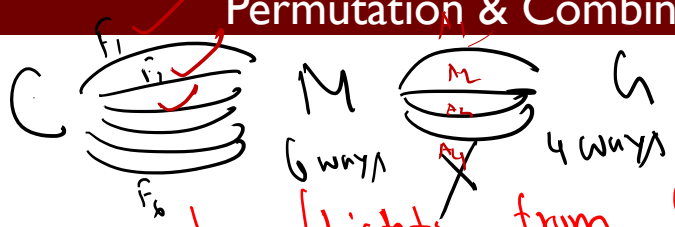
The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r

5 objects, 3 places, repetition allowed

$5 \times 5 \times 5 = 5^3$

n objects, r places $\longrightarrow n^r$

Q.1 (CSIR NET 2019)



Suppose there are 6 non-stop flights from Chennai to Mumbai in the morning and 4 non-stop flights from Mumbai to Goa in the evening. In how many ways can one fly from Chennai to Goa via Mumbai using these flights in days?

(1) 10 (2) 24 (3) 4^6 (4) 6^4

Permutations when all the objects are not distinct objects

2 X 1 = 2!

4 X 3 X 2 X 1 = 24 "ROOT" → Permutation

_ _ _ _

O O

R T

ROOT

6

5! / 3! = 5X4 = 20

O R O T

O R O T

3 X 2 X 1 = 6 = 3!

$\frac{24}{2} = 12$ different

4 letter words

different arrangements

The number of permutations of n objects, where p objects are of the same kind and rest are all different, is $\frac{n!}{p!}$

ROOT

$\frac{5!}{3!}$

Find the number of permutations of the letters of the word ALLAHABAD

A → 4

L → 2

$$\frac{9!}{4! \times 2!} = \frac{9 \times 8 \times 7 \times 5 \times 4! \times 6}{4! \times 2!} = 7560$$

36 × 35

1260

× 6

7560

Q.2 (CSIR 2016)

In how many distinguishable ways can the letters of the word CHANCE be arranged?

(1) 120

(2) 720

(3) 360

(4) 240

$\frac{6!}{2!}$

Find the number of different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that (i) all vowels occur together (ii) all vowels do not occur together

(i) D AUE G H T R — — — — — 3!

UAE

EAU 6 Objects

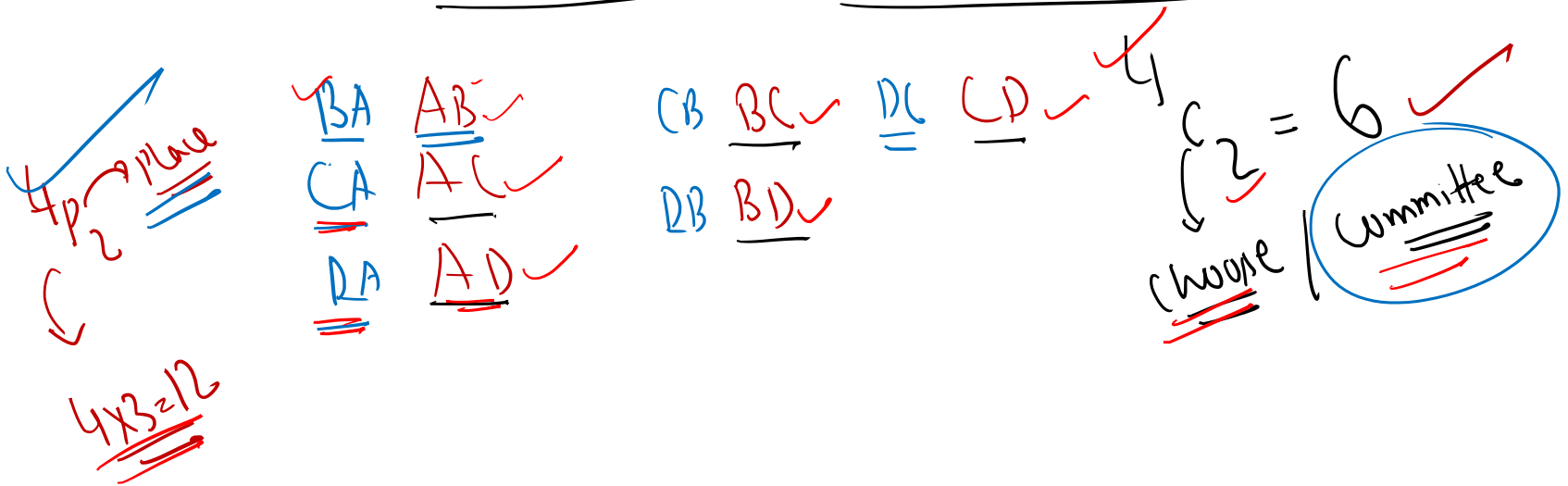
Ans = 6! × 3!

(ii) 8! − (6! × 3!) all vowels do not occur together!

8!

Combinations

Suppose we have 4 different objects A, B, C and D and we need to make combinations by taking 2 at a time.



$${}^4P_2 = 2! \times {}^4C_2 \qquad 12 = 2 \times 6$$

$$\Rightarrow {}^4C_2 = \frac{{}^4P_2}{2!}$$

$$\boxed{{}^nC_r = \frac{{}^nP_r}{r!}}$$

$$\boxed{{}^nC_r = \frac{n!}{(n-r)!\,r!}}$$

A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

$W_1\ W_2\ W_3$

$W_1 W_2$
$W_1 W_3$
$W_2 W_3$

$2M + 3W = 5$

(i) Ans $= {}^5C_3 = \frac{5!}{3!\,(5-3)!} = \frac{5\times4\times2}{\times} = 10$

(ii) Ans $= {}^2C_1 \times {}^3C_2 = \frac{2!}{1!\,(2-1)!} \times \frac{3!}{2!\,1!}$

$= 2\times3 = 6$

Anil

Bijay

${}^3C_2$

Q.3 (CSIR)

How many different salads can be made from cauliflower, tomatoes, onions, potatoes and carrots ?

(1) 16 (2) 28

(3) 31 (4) 32

Independent

$^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 =$

$$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$

$$= \frac{5!}{4!1!} + \frac{5!}{3!2!} + \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!}$$

$$= 5 + 10 + 10 + 5 + 1$$

$$= \underline{31}$$

0! = 1

(JJIR NET)

Q.4 In how many ways can you place N coins on a board with N rows and N columns such that every row and every column contains exactly one coin?

(1) N = 3

(2) N (N-1) (N-2) ... 2.1

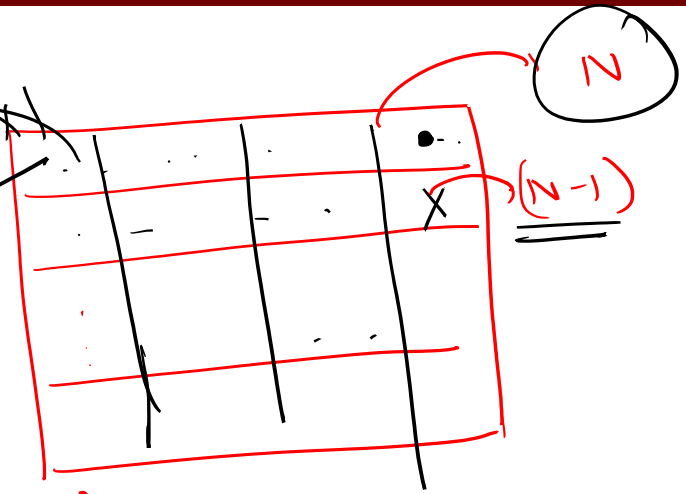
3X2X1

(3) N^2

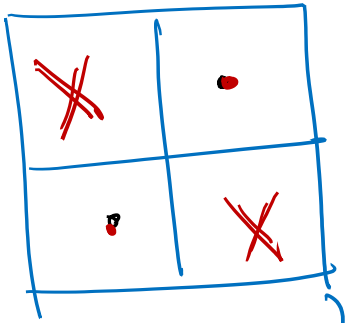
(4) N^N

N ways

$N \times (N-1) \times (N-2) \times \dots \times 2 \times 1$



N = 2

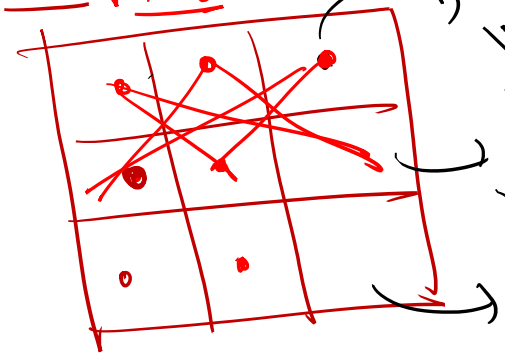


2 x 2

2 ways

2 ways

N = 3



3 x 2 = 6

3 x 2 = 6

= 6

Q.5 (CSIR) Suppose in a box there are 20 red, 30 black, 40 blue, and 50 white balls. What is the minimum number of balls to be drawn, without replacement, so that you are certain about getting 4 red, 5 black, 6 blue and 7 white balls ?

(1) 140 (2) 97 (3) 104 (4) 124

40 + 6 = 24

50 + 7 = 34

$140 - 124 = 16$

$20 - 16 = 4$

$20 + 30 + 40 + 50 = 140$

$30 - 16 = 14$

$140 - 97 = 43$

20 R

$140 - 104 = 36$

$54 - 36$

14

HAPPY LEARNING

THANKS