# GATE APTITUDE

## PROBABILITY

# Probability

Random Experiment

↓

Outcomes

↓

Sample Space

↓

Event

## Event

Any Sub collection of Sample Space.

## Examples

Throwing a Dice

↓

1, 2, 3, 4, 5, 6

↓

{1, 2, 3, 4, 5, 6}

↓

Even faces

= {2, 4, 6}

Probability ("Even face") = P("Even face")

$= \frac{3}{6} = \frac{1}{2}$

$$P(\text{event}) = \frac{n(\text{event})}{n(\text{sample space})}$$

Experiment: Tossing One coin and throwing one Dice together

Sample Space = {(H, 1), (H, 2), ...
(H, 6), (T, 1), ..
(T, 6)}

2 × 6 = 12

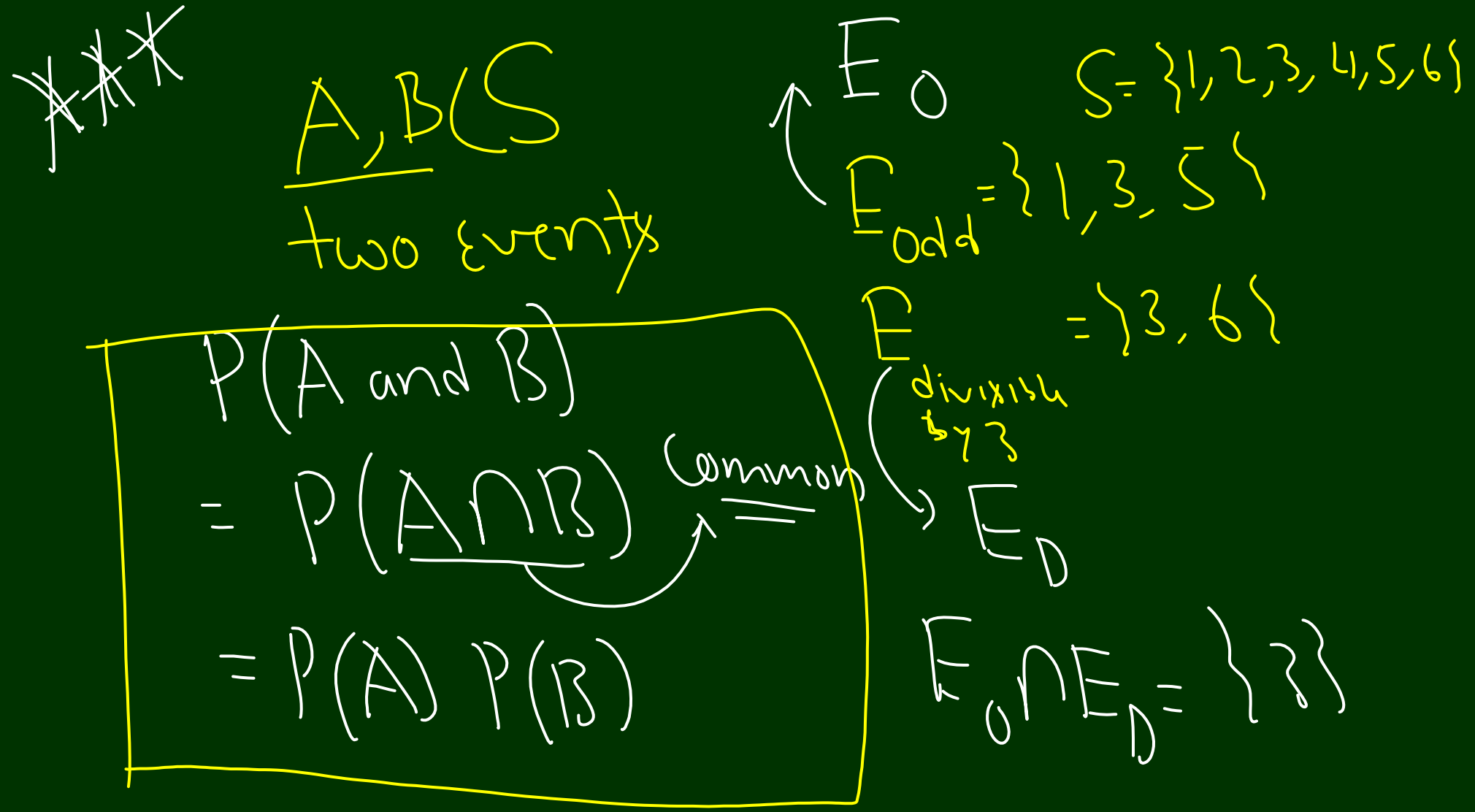
Coin Dice

$$P(\text{"head \& Odd face will come"})$$

$$= \frac{3}{12} = \frac{1}{4}$$

# Independent Events:

If two events are independent of each other, that means when probability of one event doesn't disturb the probability of other event


A, B ⊆ S
two events
P(A and B)
= P(A ∩ B)
= P(A) P(B)
Common
E
Odd
E
divisible by 3
E
D
S = {1, 2, 3, 4, 5, 6}
E = {1, 3, 5}
E = {3, 6}
E ∩ E = {3}

$$P(E_{odd}) = \frac{3}{6} = \frac{1}{2}$$

$$P(E_D) = \frac{2}{6} = \frac{1}{3}$$

$$P(E_{odd} \cap E_D) = \frac{1}{6} = \frac{1}{2} \times \frac{1}{3} = P(E_{odd})\, P(E_D)$$

$P(E_{odd}) = \frac{3}{6} = \frac{1}{2}$, $P(E_{even}) = \frac{3}{6} = \frac{1}{2}$

{1, 3, 5} ∪ {2, 4, 6}

$P(E_{odd} \cap E_{even}) = \frac{0}{6} = 0$

$E_{odd} \cap E_{even} = \emptyset$ (empty)

$\neq P(E_{odd})\, P(E_{even})$

Q.1 You are tossing two coins & What is the probability that heads will come in both?

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{1}{4}$ (4) 1

$S = \{HH, HT, TH, TT\}$

$P('HH') = \frac{1}{4}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

1st Coin  2nd Coin

$H_1$ : "1st coin head" $P(H_1) = \frac{1}{2}$

$H_2$ : "2nd coin head" $P(H_2) = \frac{1}{2}$

$$P(H_1 \cap H_2) = P(H_1)\, P(H_2)$$

and

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Q1 (CSIR NET)

A multiple choice exam has 4 questions, each with 4 answer choices Every Q has one correct answer The probability of getting all answers by independent random guesses for each one

(i) $\frac{1}{4}$ (ii) $\left(\frac{1}{4}\right)^4$ (iii) $\frac{1}{16}$ (iv) $\frac{3}{4}$

$$\checkmark \underline{Q.1} \quad \underline{Q.2} \quad \underline{Q.3} \quad \underline{Q.4}$$

$$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$$

$$= \left(\frac{1}{4}\right)^4 = \frac{1}{256}.$$

GATE 2016 IISC

Q A person moving through a tuberculosis prone zone has a 50% probability of becoming infected However only 30% of infected people develop the disease

- What % of people moving through T. prone zone remains infected but doesn't show symptoms?

(1) 15 (2) 33 (3) 35 (4) 37

$\frac{50!}{50} = \frac{50}{100} = \frac{1}{2}$

$\frac{1}{2} \times \frac{7}{10} = \frac{7}{20} = \frac{35}{100}$

$\frac{36!}{3} = \frac{3}{10}$

Infected

Symptoms

$1 - \frac{3}{10} = \frac{7}{10}$

# ** Mutually Exclusive Event

Die throw

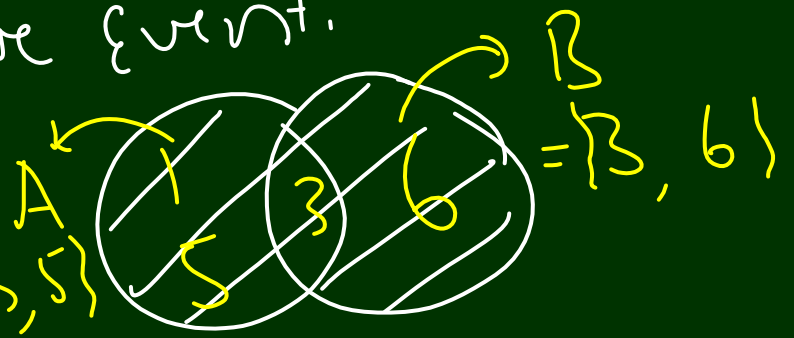
$E_{odd} = \{3, 5, 1\}$, $E_{div\ by\ 2} = \{2, 4, 6\}$

$E_{odd} \cap E_{div\ by\ 2} = \phi$.

$$P(E_{odd} \cap E_{div\ by\ 2}) = 0$$

A, B are mutually exclusive event.

$P(A \cap B) = \phi$




$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

A, B are mutually Exclusive even

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$$

$P(E_{odd}) = \frac{1}{2}$, $P(E_{even}) = \frac{1}{2}$

$P(E_{odd} \cup E_{even}) = \frac{1}{2} + \frac{1}{2}$

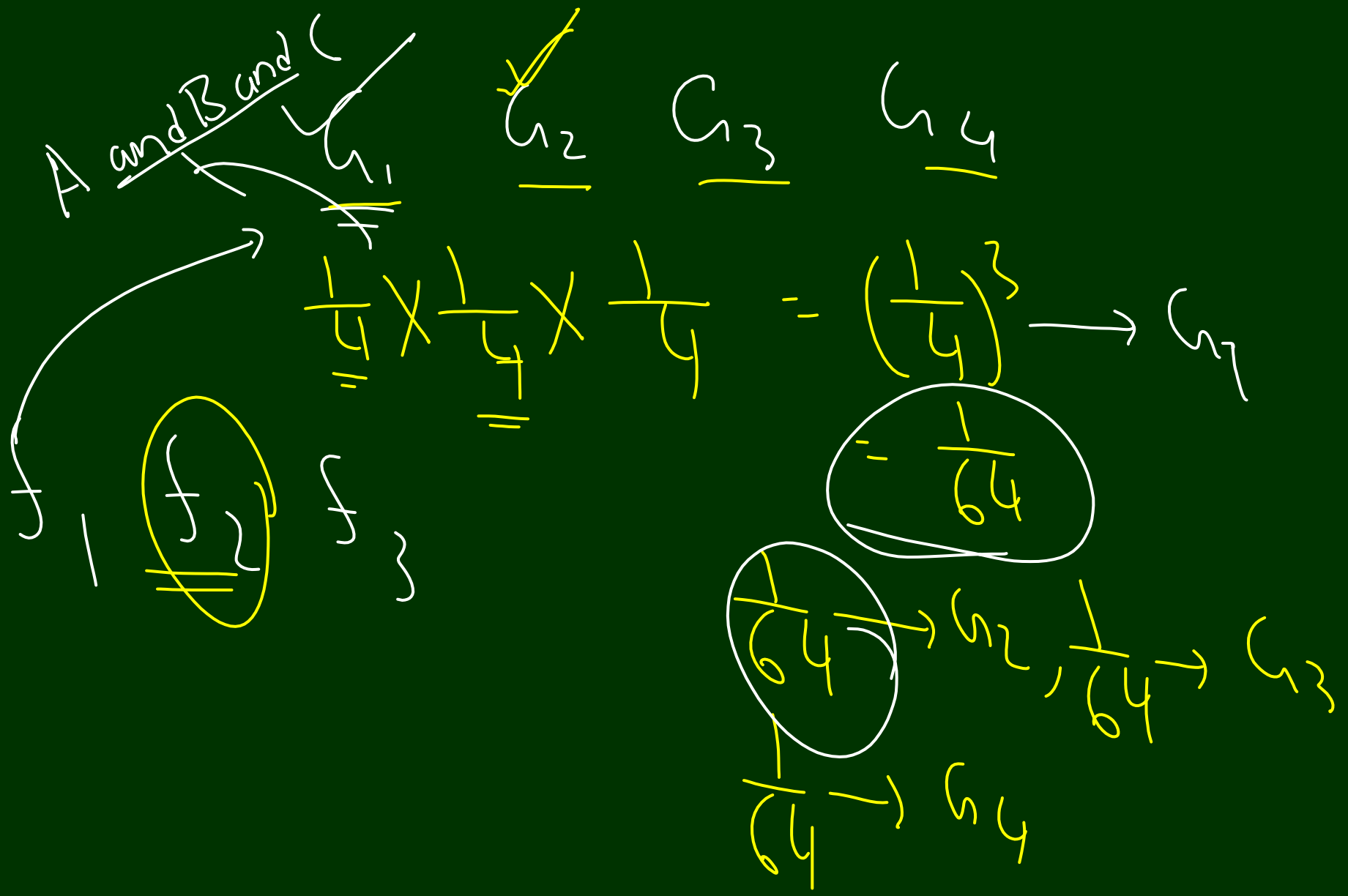
$\{1, 2, 3, 4, 5, 6\}$

$= P(E_{odd}) + P(E_{even})$

Q3 (CSIR NET)

Students of a school are divided into 4 groups what is the probability that 3 friends get into the same group?

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{1}{16}$ (D) $\frac{1}{3}$


A and B and C
$E_1$
$E_2$
$E_3$
$E_4$
$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \left(\frac{1}{4}\right)^3 \rightarrow E_1$
$\frac{1}{64} \rightarrow E_2, \frac{1}{64} \rightarrow E_3$
$\frac{1}{64} \rightarrow E_4$

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } A_4)$$

$$= P(A_1) + P(A_2) + P(A_3) + P(A_4)$$

$$= \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64}$$

$$= \frac{4}{64} = \frac{1}{16}$$

Q. (CSIR NET 2016)

The probability that a ticketless traveller is caught during a trip is 0.1. If the traveller makes 4 trips, the probability that he/she will be caught during at least one trip is

(1) $1-(0.9)^4$ (2) $(1-0.9)^4$ (3) $1-(1-0.9)^4$ (4) $(0.9)^4$

$0.1 = \frac{1}{10}$

"not getting caught"

$T_1 \quad T_2 \quad T_3 \quad T_4$

$\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

$= (0.9)^4$

"at least one of the trip getting caught"

$1 - (0.9)^4$

Q. (CSIR NET 2020)

The probability that team A wins a match against team B. is $\frac{2}{3}$. If team A and B play 4 matches against each other, what is the probability that team A will win at least one match?

FFJFJ

(1) $\frac{2}{3}$ (2) $\frac{4}{9}$ (3) 1 (4) $\frac{80}{81}$

$1 - \frac{1}{81}$ $M_1$ $M_2$ $M_3$ $M_4$

$= \frac{80}{81}$ $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

81. Loser all the match(y)

FFJFJ

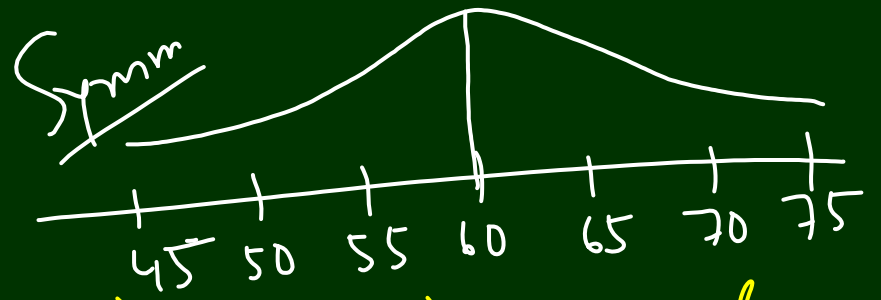
Q. (CSIR)

A fair die was thrown three times and the outcome was repeatedly six of the die is thrown again what is the probability of getting six?

(1) $\frac{1}{6}$ (2) $\frac{1}{216}$ (3) $\frac{1}{1296}$ (4) 1

FFJFJ

Q. (CSIR NOV 20)




The probability distribution of weights of a certain population is normal as shown in the figure what is the probability that the weight of the person picked at random is more than 60 kg?

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

FFJFJ

FFJFJ

HAPPY LEARNING

# THANKS